PROGRESS IN MODELING AND SIMULATION OF
SHOCK WAVE LITHOTRIPSY (SWL)

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ABSTRACT
Past research in shock wave lithotripsy (SWL) has shown that cavitation plays an essential role in the comminution of kidney stones. To provide a better understanding of the role of cloud cavitation dynamics in SWL, the flow in the focal region of a lithotripter was modeled using an ensemble averaged two-phase flow model for the bubbly mixture combined with a high-order accurate shock capturing technique. The domain and initial conditions used in the numerical model reflect the appropriate dimensions and intensity of a Dornier HM3 electrohydraulic lithotripter. The impact of factors such as the size and number of bubble nuclei in the liquid, the intensity of the shock wave and the pulse rate frequency (PRF) on the cavitating flow field is analyzed. Conclusions regarding the impact of these parameters on the potential for stone comminution are also presented.

INTRODUCTION
Several aspects of the treatment of kidney stones using SWL are not fully understood. Although the importance of cavitation in stone comminution has been firmly established, see [1-2] as well as 3 presented elsewhere in these proceedings, it has not been possible in the past to obtained detailed evolution for the generated cavitation field. Because of short and long term side effects of SWL, there is an extensive research effort to find optimized treatment parameters such as intensity and pulse rate frequency. In order to achieve this goal, it is crucial to understand the specific characteristics of the cavitation cloud which promote stone comminution. 

At the present stage, the role of numerical modeling has been constrained into predicting the pressure in the field of a lithotripter and the response of a single bubble to the pressure field [4-5]. However, these approaches are valid only in the limit of vanishing void fraction and cannot represent coupled interactions between the pressure and cavitation field. As presented in our earlier work [6-7], the two-phase continuum model is able to represent some of the complex interactions occurring in the focal region of a lithotripter. The present study is a continuation of this work and focuses on the interactions within the collapsing bubble cloud and their importance with regards to stone comminution. By post-processing the numerical results using a more complex bubble model which includes gas diffusion, we were able to calculate an approximate pulse firing rate which would correspond to the conditions implemented.

MODELING

Physical considerations
As mentioned previously, the objective of this work is to accurately model the behavior of an electrohydraulic lithotripter. The general features related to the generation and propagation of the shock wave in a lithotripter are depicted in figure 1. It should be noted that the generation of a negative pressure (tensile stress region) behind the reflector shock wave (see bottom of figure 1) has not been noted in past
experimental observations, most likely because of its small amplitude. However, this region is clearly visible in our current work and has been validated using a wave propagation model [8].

Figure 1: Shock wave propagation in an electro-hydraulic lithotripter

The geometry of the domain used in the present work is shown in figure 2. These dimensions correspond to the dimensions of the Dornier-HM3, a commercial electro-hydraulic lithotripter, and the Caltech-EHL, which is a research lithotripter based on the HM3 [5]. Typical measurements for the pressure at the focal point for two Caltech-EHL are [5]:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>APL</th>
<th>GALCIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{\text{max}}$ (MPa):</td>
<td>29.9 ± 4.7</td>
<td>35.3 ± 5.4</td>
</tr>
<tr>
<td>$P_{\text{min}}$ (MPa):</td>
<td>-11.5 ± 0.3</td>
<td>-8.7 ± 0.4</td>
</tr>
<tr>
<td>Duration (µs):</td>
<td>5-6</td>
<td>5-6</td>
</tr>
</tbody>
</table>

The magnitude of the tensile stress in the focal region is well below the vapor pressure of the liquid and generate cavitating bubbles in its wake.

Figure 2: Dimensions of the reflector used in this study

Relationship between bubble growth and duration

From the work of Lord Rayleigh [9], the time for a vapor filled spherical cavity to collapse can be derived under specific assumptions. Following the same assumptions, the time required for an impulsively forced cavitation nuclei to grow and collapse ($t_c$) can be found to be twice the Rayleigh collapse time.

$$t_c = 1.83 R_{\text{max}} \sqrt{\frac{\rho}{P_{\infty} - P_v}}$$

This simple results predicts a linear relationship between the maximum radius of a bubble and its lifetime. In figure 3, this relationship is compared to results obtained from using the Rayleigh-Plesset and Gilmore bubble model with a Church pressure waveform [10] which is the typical model used to approximate the pressure at the focal point of a lithotripter. It is interesting to note that these single bubble numerical results correspond almost exactly to the Rayleigh collapse relationship. However, when compared to measurements of the bubble size and duration in the focal region of a lithotripter [11], we find that the actual bubbles behave a significant longer lifetime for their size (see figure 3).

The discrepancy between the actual behavior of bubbles in a lithotripter and the single bubble numerical model is not caused by a limitation in the single bubble model but rather, as it will be shown in this work, by collective interaction in the bubble cloud.
Role of pulse rate frequency
Experimental studies have been conducted on the role of pulse rate frequency and have concluded that has a measurable impact on the effectiveness of the treatment [12]. Frequencies of 1 and 2Hz (which are typical rates used by clinicians) was found to be less effective in stone comminution than a rate of 1/2Hz. Bailey et al. [13] have hypothesized that the frequency dependence is due to a shielding effect of the stone by the bubble cloud. During the growth of the bubble cloud, non-condensible gas dissolves into the bubble increasing its equilibrium radius. Depending on the time delay between the arrival of the next shock wave, all or part of the trapped gases can dissolve back into solution. For a high PRF, bubbles will tend to grow to large sizes (assuming no bubble fission) and interfere to a greater extent with the focusing of the shock wave. As discussed below, this effect is in fact observed in our simulation with large bubble nuclei.

Numerical modeling of the two-phase mixture
The numerical model for the continuum bubbly flow used in this work is based on the work of Zhang and Prosperetti [14-15]. In this approach, the flow properties are average over all possible bubble location and states. In order to compute some of the ensemble average integrals, the bubble field is assumed to vary smoothly. The equations are then further simplified by assuming a low void fraction. The resulting equations show a strong similarity\(^1\) to the equations presented in the work of Biesheuvel and van Wijngaarden [16].

The derivation of the two-phase mixture equations requires the introduction of a model for the bubble dynamics. The present work follows the procedure shown in [15] in which the local perturbations in the liquid due to bubble motion are assumed to be incompressible. However, in order to provide a more realistic behavior, the model describing the bubble dynamics was corrected for compressibility (Gilmore model [17]).

The ensemble averaged equations are valid for a polydisperse bubble field. However, the cost of computing the evolution of a distribution of bubble state for all grid location is prohibitively expensive. Therefore, a monodisperse bubble distribution was used \((f(R) = f(\langle R \rangle))\).

NUMERICAL IMPLEMENTATION
The numerical implementation of our model follows closely from our previous work [6-7] and only a brief overview will be presented here.

The implementation of the ellipsoidal reflector was simplified by using a prolate-spherical coordinate system in part of the domain (see figure 4). The reflective and axisymmetric boundary condition was implemented following the work of Mohseni and Colonius [18]. The numerical implementation of the approximately non-reflective boundary conditions was based on the work of Thomson [19] and connected by adding an absorbing buffer layer based on the work of Colonius et al. [20].

In order to achieve an accurate representation of the propagation of the shock wave, a Weighted Essentially Non-Oscillatory (WENO) fifth-order shock capturing scheme was implemented. Implementation details can be found in the work of Jiang and Shu [21]. The time integration of the flow field was done using a third-order Total Variation Diminishing

\(^{1}\)The equations used for the bubbly mixture in this work the same as the one derived in [16] with the addition of a term proportional to the void fraction \(\beta_D\). They are presented in detail in [8].
(TVD) Runge-Kutta scheme [22]. The discretized ODE's describing the evolution of the bubble field were integrated separately within each time step using a fifth-order Kaps-Rentrop adaptive time marching algorithm.

**Initial conditions**

Since the shock wave generation by the spark gap is beyond the scope of this work, the simulation is initialized with the shock wave fully formed at some distance from $F_1$. This approach follows in the work of Averkiou and Cleveland [23] where the initial shock wave is taken to be of triangular profile. Because of mass conservation constraints, this waveform requires the implementation of a distributed mass source at the first focal point. This mass source is a simplified model of the vapor cavity initially generated by the firing of the electrode and is discussed in [7-8].

In addition to the uncertainty related to the initial shock wave, the conditions of the cavitation nuclei in the liquid are not known a priori. The number of bubble nuclei in suspension can be approximated from high speed images of the bubble field after the passage of the shock wave. In the work of Sokolov et al. [11], a number density of approximately 70 bubbles/cm$^3$ was reported. However, the simulation results presented in this work used a more conservative range of 0 to 40 bubbles/cm$^3$. The initial size of the cavitation nuclei cannot be determined directly from observations, but is specified and validated a posteriori. In the course of this work, sizes ranging from 3 to 50 µm were investigated and the results compared to empirical observations.

**Damage and bubble energy**

In order to identify optimal conditions for stone comminution, a method of quantifying damage must be determined. Unfortunately, no simple model is available that could approximate the damage a bubble can inflict on a surface. Alternatively, the energy released during the collapse of a bubble can be calculated. Although it is not possible to determine how much energy is released in the form of a micro-jet as opposed to a spherical shock wave or to establish a precise correlation with surface damage, we are assuming that the most energetic bubble collapse will result in the most stone damage.

In post-processing the simulation data, the energy released by a bubble was computed using a slightly different bubble model. We consider the Herring model [24]:

$$
\left[ 1 - 2 \frac{\dot{R}}{c} \right] R \ddot{R} + \frac{3}{2} \left[ 1 - \frac{4 \dot{R}}{3c} \right] \ddot{R} = \frac{p_B - p_\infty(t)}{\rho} + \frac{R}{c} \frac{dp_B}{dt} \frac{p_B - p_\infty(t)}{\rho},
$$

where $p_B$ is the pressure at the surface of the bubble. This model is nearly identical to the Gilmore model discussed earlier and has a similar behavior. However, the Herring model can be integrated once to yield:

$$
\frac{1}{2} \left( 1 + \frac{3 \dot{R}}{c} \right) R^3 \dot{R}^2 = \int_0^R \frac{p_B}{\rho} R^2 dR + \int_0^R \frac{R^3 \dot{R}}{\rho c} \frac{dp_B}{dt} dR - \frac{1}{\rho} \int_0^R \left[ p_\infty + \frac{R \dot{p}_\infty}{c} \frac{dt}{dR} \right] R^2 dR + \text{constant},
$$

The $\dot{R}/c$ dependence on the left hand side of the above equation was found to be negligible in cases relevant to this work. Similarly, the second term on the right hand side was also found to be negligible. The above equation was therefore simplified to:

$$
\frac{1}{2} R^3 \dot{R}^2 \approx \int_{R_o}^R \frac{p_B}{\rho} R^2 dR - \frac{1}{\rho} \int_{R_o}^R \left[ p_\infty + \frac{R \dot{p}_\infty}{c} \frac{dt}{dR} \right] R^2 dR,
$$

Figure 4: Representation of the numerical grid
Kinetic energy: \( \frac{1}{2} R^3 \dot{R}^2 \)

Bubble potential energy: \( \int_{R_o}^{R} \frac{p_B}{\rho} R^2 dR \)

Initial bubble energy: \( \int_{R_o}^{R} \frac{p_B}{\rho} R^2 dR \)

Work done by liquid:
\[
\frac{1}{\rho} \int_{R_o}^{R} \left[ p_\infty + \frac{R}{c} \frac{dp_\infty}{dt} \right] R^2 dR
\]

The difference between the bubble energy (internal and kinetic) and the work done by the liquid after the bubble collapse is equal to the energy released.

**Rectified diffusion**

More elaborate bubble models which can account for the contribution of heat and mass transfer in the bubble are too computationally expensive to be used in our lithotripter simulations. However, in order to be able to analyze the relationship between the size of bubble nuclei and PRF, a post-processing calculation was performed using a more elaborate bubble model in conjunction with the pressure history measured at the focal point of the simulation. A correction accounting for the internal heat and mass diffusion was introduced based on the work of Preston et al. [25]. The bubble exterior was considered incompressible and isothermal for the purpose of calculating the diffusion of non-condensible gas and its implementation followed from the work of Plesset and Zwick [26] and, Eller and Flynn [27]. It is important to note that the results shown here were based on the assumption that the liquid was degassed down to 100 Torr and the non-condensible gas is air. Changes to the actual value of concentration and/or composition of dissolved gases will impact the predictions for the PRF. Given the pressure field for a typical decoupled simulation (zero bubble number density), the required PRF to maintain the bubble equilibrium radius is shown in figure 5.

**RESULTS**

**Free field lithotripter**

Using the numerical model discussed here, the pressure and cavitation fields for a Caltech-EHL electro-hydraulic lithotripter were computed for the free field case (no stone). The predicted pressure at the focal point \( F_2 \) is compared in figure 6 to an experimental measurement obtained using a membrane hydrophone. The numerical calculations were performed for two cases: one with zero density of bubble nuclei (decoupled approach), and the other with a density of bubble nuclei of 20 bubbles/cm\(^3\). It is of interest to note that the decoupled approach provides a better match with the measurements than the case with \( N_o = 20 \) bubbles/cm\(^3\). For this case, the presence of a small pressure rise at the tail of the wave is caused by the distortion of the edge wave as it passes through the cloud of expanding bubbles [8]. This feature of the pressure wave is not usually seen in reported pressure measurements of electro-hydraulic lithotriters. The absence of this bubble—wave interaction can be partly explained by the typical experimental protocol for this measurement. In order to increase the lifetime of the pressure transducer, measurements are usually carried out in clean degassed water (low nuclei density). A second peak is some-
times observed [13] but a more detailed analysis of experimental results is needed before a more in-depth comparison can be made.

Another source of validation for the present model is the comparison of the size and shape of the calculated bubble cloud with empirical observation. Such a comparison is shown in figure 7 where appears to be in a good agreement with the observations.

As a means of finding appropriate estimates for the initial size and density of bubble nuclei, the values for the time to collapse \( t_c \) and maximum bubble radius at the focal point were compared to experimental observations. As seen in figure 8, the relationship between \( R_{\text{max}} \) and \( t_c \) is dependent on the initial conditions chosen. For cases with low or zero number density, the simulation falls very close to the Rayleigh collapse relationship. Although larger number density simulations came closer to the experimental observations, no set of initial conditions were found to match observed \( R_{\text{max}} \) and \( t_c \) simultaneously. This discrepancy may be due to the low void fraction approximation in the formulation of the ensemble averaged equations. A model capable of representing higher order interactions between the two phases may be able bridge this gap.

As seen in figure 8, the interaction between the bubble and pressure field have a significant impact on the overall behavior of the bubble cloud. The importance of this interaction is also made manifest in the collapse of the bubble cloud. Figure 9 presents the pressure and void fraction contours during the collapse of the bubble cloud. Pressure waves generated by the collapsing bubbles at the edge of the cloud propagate inward and precipitate the collapse of inner bubbles. This type of collective collapse is similar to earlier work on the collapse of spherical bubble clouds [28-29-30-31]. This phenomenon is also
shown in figure 10 where the interactions within the bubble cloud caused an 9% increase in growth and a 115% increase in the lifetime of a bubble at its center.

A direct benefit of cloud cavitation over single bubble cavitation is shown in figure 11 where the energy of a bubble at $F_2$ is compared for both cases. The energy released is estimated with equation 4 and the measured pressure at the focal point from simulation. The impact of the pressure waves observed in figure 9 on the collapsing bubbles at the center of the cloud almost tripled the amount of energy released.

The impact of the pulse rate frequency on the pressure field is illustrated in figure 12. Keeping the bubble number density constant, an increase in the PRF translates into an increase in the initial and maximum void fraction in the focal region. Because of the higher void fractions encountered, the shock wave

Figure 9: Pressure field (color contours) and void fraction (black lines) in the focal region during collapse of the bubble cloud

Figure 10: Impact of number density on bubble radius at $F_2$ for $R_o = 20\mu m$

Figure 11: Impact of number density on bubble energy at $F_2$ for $R_o = 20\mu m$
cannot focus as accurately and shielding occurs.

- for the decoupled case, the energy released increases monotonically with PRF,
- for PRF lower than 20 Hz, the energy released is increased by the presence of bubble cloud interaction,
- increasing the PRF results in an increase in $R_o$. For PRF lower than 1 Hz, this translates into an increase in void fraction, cloud interaction and energy release,
- as the PRF and $R_o$ is further increased, shielding effects due to higher void fraction decrease the pressure amplitude at focus and the energy absorbed by the bubble.

Figure 12: Peak pressure at the focal point as function of pulse rate frequency

Figure 13 presents a comparison of the energy released by collapsing bubble for a wide range of cases. The energy levels have been normalized with respect to the limiting case of zero number density and very small bubble nuclei size. Larger values of bubble number density provide significant increase in the energy release. A measure of uncertainty in the numerical results was obtained by varying an arbitrary parameter in the artificial compression method (ACM) of the WENO scheme used here (further details can be found in [8]). The data presented in figure 13 and 12 can be interpreted as follows:

- for the decoupled case, the energy released increases monotonically with PRF,
- for PRF lower than 20 Hz, the energy released is increased by the presence of bubble cloud interaction,
- increasing the PRF results in an increase in $R_o$. For PRF lower than 1 Hz, this translates into an increase in void fraction, cloud interaction and energy release,

Figure 13: Normalized energy released at bubble collapse as a function of pulse rate frequency

**Lithotripter with artificial stone**

Following the same procedure as the free field case, the impact of cloud interactions on the bubble collapse can be observed for cases with a 6.25 mm diameter artificial stone located at the focal point. Figure 14 presents the energy released by a bubble at $F_2$ in the presence of an artificial stone as a function of PRF. It should be noted that the normalization and uncertainty in figure 14 is the same as in figure 13. It is interesting to note that the same trends noticed in figure 13 appear in figure 14 and that the impact of cloud activity on the energy released is even greater in this case.
CONCLUSIONS

In this paper, we have presented some recent results using the ensemble averaged two-phase flow approach to predict cloud cavitation in SWL. The agreement between the numerical results from the present model and experimental results has been improved over our past work. Using the energy analysis presented here, we were able to observe how the collapse of the bubble cloud can generate a substantial increase in the energy available for bubble collapse. Moreover, by post-processing simulation results using a more complex bubble model with gas diffusion, initial simulation conditions were related to the pulse rate frequency. Based on the results presented here, the following conclusions can be drawn:

- the discrepancy in the ratio of $t_c/R_{\text{max}}$ between direct observation and single bubble models can be explained by bubble interactions in the cloud,

- subjected to the same pressure history, the energy released increases with the bubble equilibrium size (PRF),

- for non-zero bubble number density, the collapse of the bubble cloud generates pressure wave which enhances the energy released by the inner bubbles by up to a factor of three,

- higher void fractions deflect part of the shock wave energy away from the focal point and prevent inner bubbles from acquiring as much energy,

- based on the previous observations, a PRF of approximately 1 Hz provides conditions for the maximum release of energy by a bubble at the focal point and presumably inflicts the maximum stone damage for a given number density.

It should be kept in mind that the values for the PRF are dependent on the initial fraction of gas dissolved in the liquid. Furthermore, it should be noted that, if present, bubble fission would increase the number density and decrease the equilibrium size. There is some evidence that this in fact takes place [32]. At this stage, this phenomena cannot be introduced into the model and any comparison of the PRF results with experimental observations should be made for cases with a stable density of bubble nuclei.

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REFERENCE


